Comment on "Wealth condensation in Pareto macroeconomies"

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In a recent study of the Pareto macroeconomy [Phys. Rev. E **65**, 026102 (2002)], a surprising deviation to the power law distribution of the large wealths is reported. We comment that such a "corruption" phenomenon can be reproduced in a much simplified framework. The corruption disappears when the small wealths are further included in a mean-field treatment. The constraint of the total-wealth conservation leads to a cutoff in the power-law tail, in contrast to the prominent enhancement reported previously.

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In a recent publication [1], the Pareto macroeconomies are discussed with a sophisticated treatment of the Levy partition function. A significant deviation to the expected power-law Pareto tail is observed in the social phase of a closed system with fixed total wealth [ensemble (a) of Ref. [1]]. When the average wealth is larger than a critical value, the power-law distribution of large wealths is terminated with a prominent peak (see Fig. 1 of Ref. [1]). The emergence of this peak implies that a sizable fraction of the total wealth is amassed by a single individual. Such an interesting wealth condensation in the Pareto macroeconomies is thus named the "corruption" phenomenon.

We comment that the results can be reproduced within a much simplified framework. Consider a normalized power-law distribution as follows:

$$P(w)=\frac{\alpha}{w^{1+\alpha}}, \tag{1}$$

where the exponent $\alpha>1$ and $w\in(1,\infty)$ denotes the wealth. For such a distribution, the average wealth can be easily calculated as $\alpha/(\alpha-1)$. With an ensemble of size N, let w_i denote individual's wealth (with index $i=1,\ldots,N$) and the average wealth $\overline{w}=(1/N)\Sigma_{i=1}^{N}w_i$. If we draw w_i directly from the above power law distribution and impose the conservation of total wealth, the corruption phenomenon is reproduced, see the symbol ($\times$) in Fig. 1. We note that whenever the average wealth is assigned a value larger than $\alpha/(\alpha-1)$, the corruption phenomenon emerges. This feature can be retrospected to the presumption of Eq. (1).

In practice, the power-law distribution is expected to be valid only for the large wealths. Within an economy, however, the large wealths cannot be isolated from the small wealths. To take into account both the large wealths and the small wealths, we consider a simple mean-field treatment with the following stochastic equations [2]:

$$\frac{dw_i}{dt}=\eta_i(t)\,w_i+J(\overline{w}-w_i). \tag{2}$$

The spontaneous fluctuation is prescribed by a multiplicative random source $\eta_i(t)$. The trading between individuals is controlled by the parameter J [3]. For large wealths, the power-law distribution is well reproduced,

$$P(w)\propto\frac{1}{w^{1+\alpha}}, \tag{3}$$

where the exponent $\alpha=1+J/\sigma^2>1$ and $2\sigma^2$ is the variance of the random variable $\eta_i(t)$. For small wealths, the distribution is properly suppressed as compared to Eq. (1).

Within the stochastic multiplicative model of Eq. (2), the total wealth of an economy is not conserved. The growth of the total wealth is controlled by the noise $\eta_i(t)$. The average total wealth can be kept constant by setting $m+\sigma^2=0$, where m denotes the mean of the stochastic noise. In the thermodynamic limit, i.e., $N\to\infty$, a steady distribution is obtained [2] (shown by the solid line in Fig. 1),

$$P(a)\propto\frac{\exp[-(\alpha-1)/a]}{a^{1+\alpha}}, \tag{4}$$

where $a\equiv w/\overline{w}$ denotes the normalized wealth. It is interesting to note that the above distribution is independent of both m and $\overline{w}$, i.e., the variation of the average wealth will not affect the distribution. The effect of a finite N leads to a cutoff in the power-law tail [4]. As the individual wealth is limited by the total wealth, the power-law tail of the distri-

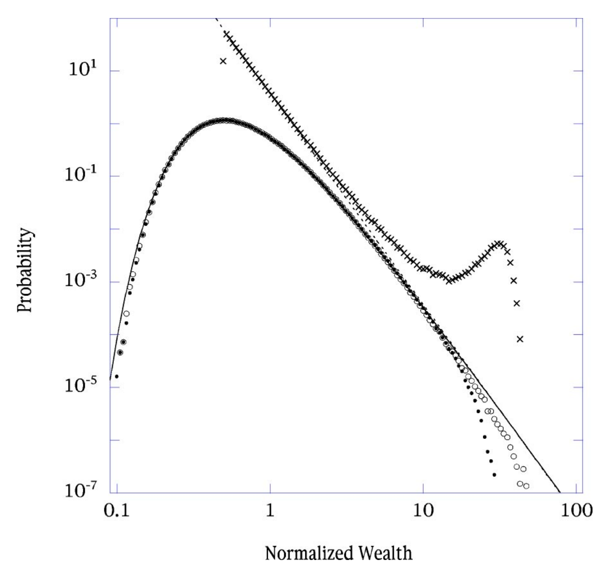

FIG. 1. Wealth distribution $P(a)$ for the normalized wealth $a=w/\overline{w}$. Parameters are $\alpha=3$, $\overline{w}=2$, and $N=128$. The corruption phenomenon can be reproduced when the small wealths are neglected; the results are shown by the symbol ($\times$). For the mean-field model of Eq. (2), the results with and without total-wealth restriction are shown by the symbols (●) and (○), respectively. The solid line shows the analytic results in the thermodynamic limit. The dashed line shows the extrapolation of the power-law tail.

bution $P(a)$ is truncated, see the symbol (○) in Fig. 1. To examine the effect of total-wealth conservation, further constraint has to be imposed. We find that such a constraint leads to a stringent cutoff to the power-law tail. The results are shown by the symbol (●) in Fig. 1.

In this Comment, we show that the corruption emerges when the small wealths are totally neglected and the large wealths described by the power-law distribution are taken as an isolated economy. When the total-wealth conservation is imposed properly on the dynamic equations governing both small and large wealths, a stronger cutoff to the power-law distribution is presented. The corruption phenomenon is absent from the Pareto macroeconomy described by the stochastic multiplicative model.

[1] Z. Burda, D. Johnston, J. Jurkiewicz, M. Kamiński, M.A. Nowak, G. Papp, and I. Zahed, Phys. Rev. E **65**, 026102 (2002).

[2] J.-P. Bouchaud and M. Mézard, Physica A **282**, 536 (2000).

[3] In the mean-field approximation, the redistribution parameters assume the same value, i.e., $J_{ij}=J/N$. When J_{ij} are allowed to fluctuate around this mean-field value, the small wealths are enhanced and the distribution $P(w)$ can be extended to $w<0$; the distribution for large wealths are basically the same. And we obtain the same conclusion of no corruption.

[4] Z.F. Huang and S. Solomon, Physica A **294**, 503 (2001).